Problem 1: An Impossible Electric Field

One of these vector fields cannot possibly be an electric field. Which one?

(a) $\vec{E} = \alpha\, y\, z\, \hat{x} + \alpha\,(z\,x - 4\,z\,y)\hat{y} + \alpha\,(x\,y - 2\,y^2)\,\hat{z}$.

(b) $\vec{E} = -3\,\alpha\,x\,z\,\hat{x} + \alpha\,x\,y\,\hat{y} + 2\,\alpha\,y\,z\,\hat{z}$

Here α is a constant with appropriate units. For the $\vec{E}$ which could be an electric field, find the potential at a point with coordinates (x, y, z). Use the *origin* as the reference point and find V by integrating $d\vec{\ell} \cdot \vec{E}$ along any path that starts at the origin and ends at the point (x, y, z). Check that $-\vec{\nabla}V$ for your answer gives the correct $\vec{E}$.

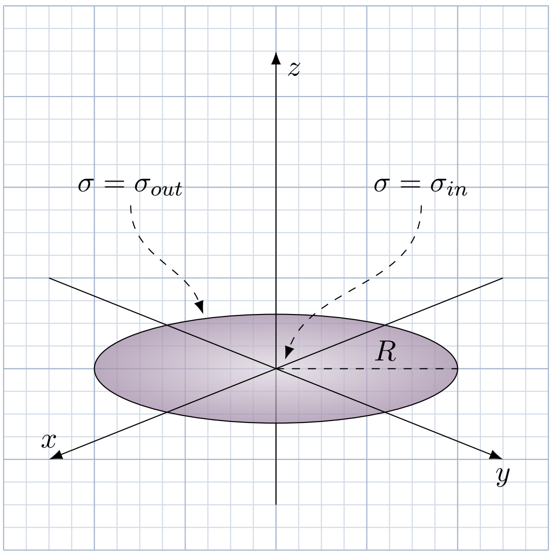
> HINT: You can use any path between the origin and (x, y, z) to find V – they will all give the same result – but to perform the integral you must choose *some* path. Use one that makes the integral easy. For instance, you could integrate along the x-axis from $(0, 0, 0)$ to $(x, 0, 0)$, then in the y-direction from $(x, 0, 0)$ to $(x, y, 0)$, and so on. Or you could draw a straight line from the origin to the point and integrate along that: $(x', y', t') = (x\,t, y\,t, z\,t)$ with $0 \le t \le 1$. In that case, $d\vec{\ell}\,' = x\,dt\,\hat{x} + y\,dt\,\hat{y} + z\,dt\,\hat{z}$, which you would dot into $\vec{E}(x\,t, y\,t, z\,t)$.

Problem 2: Non-Uniform Surface Charge on a Disk

Find the electric potential at a point above the center of a disk of radius R with a surface charge density that changes linearly from σ_{in} at the center of the disk σ_{out} at its outer edge:

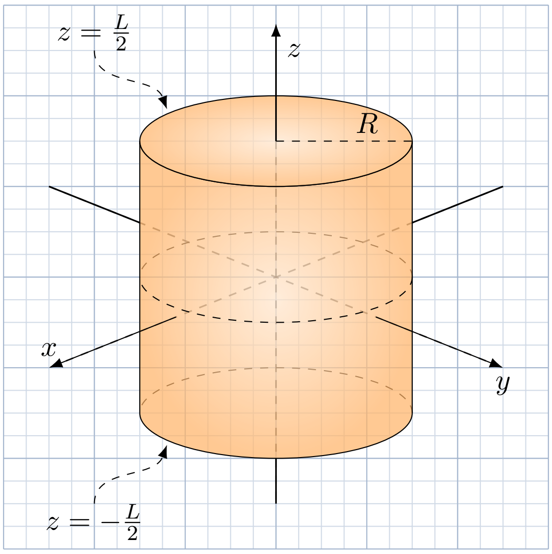
$$\sigma(s) = \sigma_{in} + \frac{s}{R}\left(\sigma_{out} - \sigma_{in}\right),$$

where σ_{in} and σ_{out} are constants, and s is the distance from the center of the disk. To make things easy, assume that the disk sits in the x-y plane, with the origin at the center of the disk, and use cylindrical polar coordinates. You may not use a computer to perform the integral. Either evaluate it on your own or use an integral table. If you use an integral table, provide a reference at the end of your solution.



Problem 3: Potential due to a Uniform Charged Cylinder

A solid cylinder of length L and radius R carries a uniform volume charge density ρ_0. Find the potential at a point on the axis of the cylinder, a distance z from its center. Assume that $z > L/2$ so the point is outside of the cylinder.



*Problem 4: Potential due to a Solid Uniform Sphere

You already know that the electric potential outside a uniform charged solid sphere of radius R and total charge Q is the same as a point charge. Calculate the electric potential *inside* a uniformly charged solid sphere using the formula

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} d\tau' \, \frac{\rho_0}{\mathscr{r}} \ ,$$

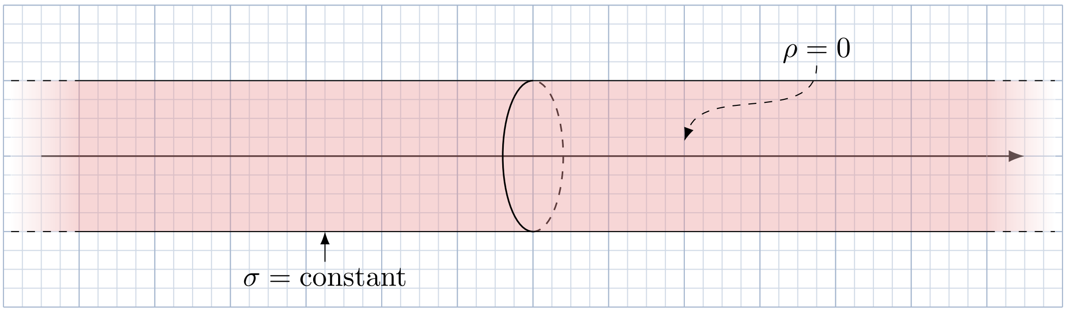
where $\rho_0 = Q/(\frac{4}{3}\pi R^3)$ is the constant charge density. You can check your answer by taking the $R_i \to 0$ limit of the electric potential for a spherical shell of inner radius R_i and outer radius R, which we worked out in class. Finally, use the gradient of your answer to find the electric field inside the sphere.

> HINT: The set-up here is just like the spherical shell example from class. Make sure your z-axis is in the direction of $\vec{r}$, so that $\vec{r} = r\,\hat{z}$. Set up the integrand the same way, and remember that, since $0 \le r \le R$, the integral over r' will involve regions where $0 \le r' < r$ and regions where $r < r' \le R$. In the former, $\sqrt{(r-r')^2} = r - r'$. In the latter, it's $\sqrt{(r-r')^2} = r' - r$.

Problem 5: Gauss' Law for a Long, Hollow Cylinder

Use Gauss' law to find the electric field inside and outside a long, hollow, cylindrical tube of radius R which carries a uniform surface charge density σ. Check that your result agrees with the expected discontinuity in the component of the electric field normal to the surface:

$$\vec{E}_{out}(s = R) - \vec{E}_{in}(s = R) = \frac{\sigma}{\epsilon_0}\,\hat{n}$$


$\rho = 0$
$\sigma =$ constant