## **Problem 1: An Impossible Electric Field**

One of these vector fields cannot possibly be an electric field. Which one?

- (a)  $\vec{E} = \alpha y z \hat{x} + \alpha (z x 4 z y) \hat{y} + \alpha (x y 2 y^2) \hat{z}$ .
- (b)  $\vec{E} = -3 \alpha x z \hat{x} + \alpha x y \hat{y} + 2 \alpha y z \hat{z}$

Here  $\alpha$  is a constant with appropriate units. For the  $\vec{E}$  which could be an electric field, find the potential at a point with coordinates (x, y, z). Use the *origin* as the reference point and find V by integrating  $d\vec{\ell} \cdot \vec{E}$  along any path that starts at the origin and ends at the point (x, y, z). Check that  $-\vec{\nabla}V$  for your answer gives the correct  $\vec{E}$ .

**HINT:** You can use any path between the origin and (x, y, z) to find V – they will all give the same result – but to perform the integral you must choose *some* path. Use one that makes the integral easy. For instance, you could integrate along the x-axis from (0,0,0) to (x,0,0), then in the y-direction from (x,0,0) to (x,y,0), and so on. Or you could draw a straight line from the origin to the point and integrate along that: (x', y', t') = (x t, y t, z t) with  $0 \le t \le 1$ . In that case,  $d\vec{\ell}' = x dt \hat{x} + y dt \hat{y} + z dt \hat{z}$ , which you would dot into  $\vec{E}(x t, y t, z t)$ .

### Problem 2: Non-Uniform Surface Charge on a Disk

Find the electric potential at a point above the center of a disk of radius R with a surface charge density that changes linearly from  $\sigma_{in}$  at the center of the disk  $\sigma_{out}$  at its outer edge:

$$\sigma(s) = \sigma_{in} + \frac{s}{R} \left( \sigma_{out} - \sigma_{in} \right),$$

where  $\sigma_{in}$  and  $\sigma_{out}$  are constants, and s is the distance from the center of the disk. To make things easy, assume that the disk sits in the x-y plane, with the origin at the center of the disk, and use cylindrical polar coordinates. You may not use a computer to perform the integral. Either evaluate it on your own or use an integral table. If you use an integral table, provide a reference at the end of your solution.



# Problem 3: Potential due to a Uniform Charged Cylinder

A solid cylinder of length L and radius R carries a uniform volume charge density  $\rho_0$ . Find the potential at a point on the axis of the cylinder, a distance z from its center. Assume that z > L/2 so the point is outside of the cylinder.



### \*Problem 4: Potential due to a Solid Uniform Sphere

You already know that the electric potential outside a uniform charged solid sphere of radius R and total charge Q is the same as a point charge. Calculate the electric potential *inside* a uniformly charged solid sphere using the formula

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} d\tau' \frac{\rho_0}{\imath} \, ,$$

where  $\rho_0 = Q/(\frac{4}{3}\pi R^3)$  is the constant charge density. You can check your answer by taking the  $R_i \to 0$  limit of the electric potential for a spherical shell of inner radius  $R_i$  and outer radius R, which we worked out in class. Finally, use the gradient of your answer to find the electric field inside the sphere.

**HINT:** The set-up here is just like the spherical shell example from class. Make sure your z-axis is in the direction of  $\vec{r}$ , so that  $\vec{r} = r \hat{z}$ . Set up the integrand the same way, and remember that, since  $0 \le r \le R$ , the integral over r' will involve regions where  $0 \le r' < r$  and regions where  $r < r' \le R$ . In the former,  $\sqrt{(r-r')^2} = r - r'$ . In the latter, it's  $\sqrt{(r-r')^2} = r' - r$ .

### Problem 5: Gauss' Law for a Long, Hollow Cylinder

Use Gauss' law to find the electric field inside and outside a long, hollow, cylindrical tube of radius R which carries a uniform surface charge density  $\sigma$ . Check that your result agrees with the expected discontinuity in the component of the electric field normal to the surface:

$$\vec{E}_{out}(s=R) - \vec{E}_{in}(s=R) = \frac{\sigma}{\epsilon_0}\,\hat{n}$$

